Self lensing effects for compact stars and their mass-radius relation

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Abstract

During the last couple of years astronomers and astrophysicists have been debating on the fact whether the so called 'strange stars' - stars made up of strange quark matter, have been discovered with the candidates like SAX J1808.4−3658, 4U 1728−34, RX J1856.5−3754, etc. The main contention has been the estimation of radius of the star for an assumed mass of $\sim 1.4 M_{\odot}$ and to see whether the point overlaps with the graphs for the neutron star equation of state or whether it goes to the region of stars made of strange matter equation of state. Using the well established formulae from general relativity for the gravitational redshift and the 'lensing effect' due to bending of photon trajectories, we, in this letter, relate the parameters M and R with the observable parameters, the redshift z and the radiation radius R_{∞}, thus constraining both M and R for specific ranges, without any other arbitrariness. With the required inputs from observations, one ought to incorporate the effects of self lensing of the compact stars which has been otherwise ignored in all of the estimations done so far. Nonetheless, these effect of self lensing makes a marked difference and constraints on the M-R relation.

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With the new observations coming in from the satellites Chandra and XMM Newton, it is getting trickier to decide whether many of the X-ray

emitting stars observed are just neutron stars or more exotic stars made up of strange quark matter [1, 2, 3]. The equilibrium configuration for stars with matter other than just nucleons, need a very compact structure and thus with the masses of around $1 \sim 1.5M_{\odot}$, they have to be far more smaller and compact than conventional neutron stars. There have been several discussions on strange stars but the community of astrophysicists do not seem to have any final say on this, with several groups differing in opinion on same candidates. However, it is clear that in spite of difficulties involved, astronomers are being more and more successful in getting lot more details for many candidates, particularly for the ones having X-ray emission. Apart from getting into debates over the issue, it is perhaps quite necessary to consider the theoretical constraints that the presently accepted physical theories, yield purely from logical reasons. One needs to consider basically the mass-radius relation which arises from different possible effects that the observations are constrained with.

As general relativity has been accepted as the most successful theory to describe gravity, it is necessary to consider seriously the effects it brings in while estimating parameters of the stellar structure. One of the constraint which everyone seem to accept gracefully is the gravitational redshift factor, while observing a distant massive object, as most of the discussions relate the so called *observed radius* R_∞ (Radiation radius) with the actual radius R, as given by

$$R_\infty = RA^{-1} \tag{1}$$

A being the redshift factor, given by $A = (1-2m/R)^{1/2}$ for the Schwarzschild metric representing the field of a static star, while $A = (1 - 2m/R - \omega^2 R^2 Sin^2\theta)^{1/2}$ for the linearised Hartle-Thorne metric [4], representing the field of a slowly rotating star with angular velocity ω, which is given by

$$\begin{aligned} ds^2 = (1-\frac{2m}{r})dt^2 - (1-\frac{2m}{r})^{-1}dr^2 - r^2 d\theta^2 \\ -r^2 Sin^2\theta(d\phi - \omega dt)^2 \end{aligned} \tag{2}$$

where $m = GM/c^2$ represents the mass M in geometrical units. Equation (1) thus yields for nonrotating star, the equation

$$R^3 - R_\infty^2(R-2m) = 0 \tag{3}$$

and the equation

$$R^6 - R_\infty^2(R^4 - 2mR^3 - 4J^2 Sin^2\theta) = 0 \tag{4}$$

for the rotating star, where J is the specific angular momentum related to ω, through the relation $\omega = 2J/R^3$. With these two, one can easily workout the actual radius R of the star in either case, for a given M and R_∞ the observed radiation radius.

However, it is to be remembered that general relativity predicts another important effect, that of gravitational lensing, associated with bending of light rays passing across the star. In the case of extremely compact objects, the gravitational potential M/R is so large that there could be self lensing effect, where the radiation coming from the near neighbourhood of a compact star would get lensed such that the star from a distance appear much bigger than what it actually is. Nollert et al. [5] considered this effect in analyzing the 'relativistic looks' of a neutron star and have graphically depicted the consequences of the relativistic light deflection. As they point out if I_{ν_∞} is the observed specific intensity by an asymptotic observer, then it is related to the intrinsic intensity I_{ν_s}, through the relation

$$I_{\nu_\infty} = I_{\nu_s}\nu_\infty^3/\nu_s^3 = I_{\nu_s}A^3 \tag{5}$$

where A is again the redshift factor given earlier. As $I_\nu \propto F/R^2$, F being the total flux of radiation, one can easily find that, when self lensing effect is taken into account, the two radii R_∞ and R are related through the equation

$$R_\infty^2 = R^2 A^{-3} \tag{6}$$

It is to be noted that there can be some variation in the observed flux, but considering an equilibrium state of the star, it can be taken to be constant for a smaller time scale (of observation). Using the expression for A, the relation between R_∞ and R turns out to be

$$R^7 - R_\infty^4(R^3 - 6mR^2 + 12m^2R - 8m^3) = 0 \tag{7}$$

for the non rotating case and

$$\begin{aligned} R^{10} - R_\infty^4(R^6 - 6mR^5 &+ 12m^2R^4 + 8m^3R^3 \\ &- 12J^2Sin^2\theta(R-2m)^2) = 0 \end{aligned} \tag{8}$$

for the rotating case, wherein the powers of J^4 are neglected. With these equations it is clear that given a R_∞ and J, there is no guarantee that real positive roots exist for R for any M. Fig. (1) shows the plots of M vs R for different values of R_∞ for the case with rotation ($j = 0.3m^2$). The plots

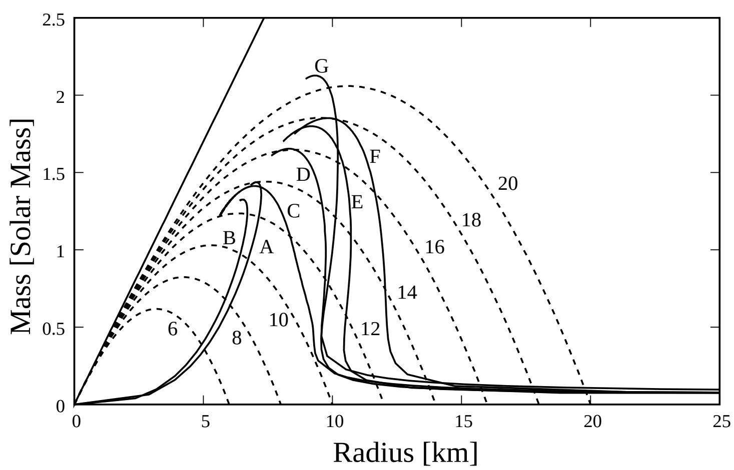

Figure 1: The dashed curves, labeled as 6, 8, 10, etc. (up to 20, in units of km), show the allowed variation of mass with the radius for different values of the radiation radius. The two lines labeled A & B are from the strange star equation of states. The curve labeled C is for the hyperon star and the curves labeled D, E, F and G are for different neutron star equation of states. The straight line is the line for the event horizon.

for the case $j = 0$ hardly differs from these curves as the effects appear only with j^2 order.

Another candidate which can be taken as an example is the compact star PSR B0656+14 [11]. It has been studied in various wavelengths, from optical [12] to x-rays [13]. Two component blackbody models are commonly used in measuring the surface temperature of the star. The hard component of the blackbody spectrum is identified with the hot polar caps and the soft component is due to the general photospheric radiation and is directly related to the stellar surface temperature. The redshifted surface temperature (T_∞) is related to the temperature of the blackbody fit by

$$T_\infty = \frac{T}{1+z} \tag{9}$$

From ROSAT data, Koptsevich et al.[12] estimated a value of $\mathrm{T}_\infty \sim 8\times10^5$K and quite independently Marshall and Schultz [13], from Chandra observations gave similar results. This value of T_∞ gives in turn, the radiation radius (R_∞) of the stellar photosphere for the assumed distance estimate of 288 pc. Said so, with this value, R_∞ is calculated to be $\sim$ 8 km. This immediately points the source to be a very compact strange star. So, the believers of neutron star models immediately *corrected* themselves, saying that the star's temperature has been overestimated, and hence should be changed to a lower value in order to give a larger value of R_∞, that can go very well with the normal neutron stars. These types of guesses are always made to keep a star in the conventional neutron star regime. However, none of the calculations make use of the lensing effect that is more concrete. Even with a large range of guess values for the surface temperature, and subsequently a large range of values in R_∞, the effect of lensing (Eq. 7) allows a smaller window for the M-R curve, than that without lensing taken into consideration.

Recently there have been attempts to measure redshift of spectral lines emitted from regions close to the stellar surface [6, 7] and using this information they obtain the radius R for an assumed mass M. However, if both the observational data regarding the radiation radius R_∞ and the redshift z are to be taken seriously, then one can work out both the radius R and mass M of the star uniquely using equation (6) and the relation

$$1 + z = (1 - 2m/R)^{-1/2} \tag{10}$$

as given by

$$R = (1+z)^{-3/2} R_\infty \tag{11}$$

and

$$m = \frac{z(z+2)}{2(1+z)^{7/2}} R_\infty. \tag{12}$$

Fig. (1) gives the plots of $m(R)$ for given R_∞ values as well as the plots for various theoretical compact star models made from various equation of state of the matter. From the plot it is clear that for a given value of R_∞, there is a maximum limit of the mass up to which the stars can exist with real mass-radius relation. The straight line represents the event horizon for each mass, which is a natural bound for the star not to be a black hole. Prasanna and Ray [8] had considered earlier this case for the star RX J1856.5−3754 [9, 10] which had created a lot of controversies regarding its nature, with the estimated radiation radius for this X-ray star being highly uncertain, ranging from 8km to 15km. It is now obvious from the figure that for these values of R_∞ the maximum allowed mass range is between 0.8 and 1.5 $M_\odot$. As one does not have the redshift measurement for this, one can only guess and as the overlapping curves show, the star could be either a neutron star or a strange star. Hence it is very clear that with the existing observational evidence, one cannot rule out the possibility of the star RX J1856.5−3754 being a strange star. However, one would say that it is premature to come to conclusions one way or the other unless one has both the photometric (for the radiation radius) and the spectrometric (for the redshift) observations for these compact objects.

This effect of lensing is nothing new, and has been discussed in many context, but has been ignored in all the calculations for evaluating the M-R relations of the compact stars. However, as we showed, with this effect taken into account, they impose more constraints in the M-R window, and hence can give a better picture of the compact stellar candidates.

References

[1] M. Dey, I. Bombaci, J. Dey, S. Ray & B. C. Samanta, Phys. Lett. B438, (1998) 123; erratum B467, (1999) 303.

[2] X-D. Li, I. Bombaci, M. Dey, J. Dey & E. P. J. van den Heuvel, Phys. Rev. Lett., 83, (1999a) 3776.

[3] X-D. Li, S. Ray, J. Dey, M. Dey & I. Bombaci, ApJ, 527, (1999b) L51.

[4] J. B. Hartle & K. S. Thorne, ApJ, 153, (1968) 807; ApJ, 158, (1969) 719.

[5] H. P. Nollert, H. Ruder, H. Herold & U. Kraus, A&A, 208, (1989) 153.

[6] J. Cottam, F. Paerels & M. Mendez, Nature, 420, (2002) 51.

[7] D. Sanwal, G. G. Pavlov, V. E. Zavlin & M. A. Teter, ApJ, 574, (2002) L61.

[8] A. R. Prasanna & S. Ray, "Strange stars, have they been discovered", Nova Publications, (in press) (2004).

[9] J. J. Drake *et al.*, ApJ, 572, (2002) 996.

[10] F. M. Walter & J. M. Lattimer, ApJ, 576, (2002) L145.

[11] W. F. Brisken, S. E. Thorsett, A. Golden & W. M. Goss, ApJ, 593, (2003) L89.

[12] A. B. Koptsevich *et al.*, A&A, 370, (2001) 1004.

[13] H. L. Marshall & N. S. Schultz, ApJ, 574, (2002) 377.